

## MTH 234 - Quiz 4

Due 19 June at the beginning of class

You may work together on solving these problems, but what you turn in must be your own work written in your own words; copying another person's work is not allowed. Please present your work in a clear and organized fashion, and staple your pages (if needed). Late quizzes will receive at most 75% credit if they are turned in by the middle of class; after that, no more quizzes will be accepted.

1. (5 points) Find the volume of the solid in the first octant described by the inequalities

$$1 \leq x^2 + y^2 + z^2 \leq 4$$
$$\sqrt{x^2 + y^2} \leq z \leq \sqrt{3}\sqrt{x^2 + y^2}$$

2. (5 points) A tetrahedron has vertices  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(0, 2, 0)$  and  $(0, 0, 3)$ ; the density of the tetrahedron at a point is proportional to the height above the  $xy$ -plane. Find its mass and center of mass.
3. (5 points) Compute and sketch the gradient field of the function  $f(x, y) = x(x + y)$ .
4. (5 points) Suppose that  $C$  is a smooth curve given by a vector function  $\vec{r}(t)$ ,  $a \leq t \leq b$ . Show that

$$\int_C \vec{r} \cdot d\vec{r} = \frac{1}{2} [|\vec{r}(b)|^2 - |\vec{r}(a)|^2].$$